

JOURNAL OF CYCLE RESEARCH

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EDITORIAL

The talk by John Nelson relative to radio weather forecasting, which constitutes the lead article of this issue, epitomizes 12 years of work by Nelson at RCA Communications plus considerable overtime at home. It is a most important document.

But from the standpoint of cycle science it is only the beginning. What we now need is statistical verification of Nelson's observations. Are these correspondences that he has observed mere coincidences, or are they statistically significant?

It is hard to see how Nelson could attain his enviable forecasting record if his methods are fallacious. On the other hand, there are no known physical justifications for the behavior he has observed.

No one who knows Nelson would question his sincerity or his integrity. Moreover he speaks with the authority of one who has seen his forecasts come true consistently. And finally there is the excellent accuracy record, year after year. Yet the physicist is warranted in saying, "These things cannot be." Or at least he would be justified in so saying if he had not learned during the last few years that what "cannot be" today is a commonplace of tomorrow.

As Nelson would be the first to admit, proof is needed, one way or the other. If Nelson is wrong and his correspondences are mere coincidences, the sooner we know it, the better. But if he is right, he is entitled to be credited with discoveries of revolutionary importance.

The Foundation for the Study of Cycles has set itself the task of finding out the truth of this matter.

Edward R. Dewey, Director

THE CYCLIC ELEMENTS OF RADIO WEATHER FORECASTING¹

BY JOHN H. NELSON²

MY COMPANY, RCA Communications, is, in my estimation, the most efficient world-wide communication organization in existence today. We hold that strong position through the fact that we are interested in basic research. The application of this basic research by well-trained personnel, goes throughout the whole organization.

There are several definite cycles in our business that require study. Three of them are tied to the sun.

The first such cycle is of course the daily cycle. We have to change frequencies throughout the day on our various radio circuits to match the strength of the earth's ionosphere which varies, depending upon the amount of sunlight which falls upon it, at different times of the day.

Second, there is a seasonal effect in the strength of the ionosphere which we must match. In winter time, the frequencies required are entirely different from the frequencies required at other times of the year. If we do not match these frequency requirements we can get into trouble. So there is a seasonal cycle which changes month by month.

Third we have the famous 11-year sunspot cycle. This is probably the most important to us on a long range basis because we must plan a long time ahead just what frequencies are going to be needed, what types of antenna are going to be required to handle these frequencies, and how much power is going to be needed on our various circuits. And since we have the entire world tied together by radio, it's important. All these radio circuits require slightly different frequencies; some are vastly different.

To give you an example of how our frequencies vary throughout the 11-year sunspot cycle, I wish you would look at Fig. 1, which shows you how the frequency requirement cycle between New York and Central Europe changes with the sunspot cycle.

If we do not match what you see on this graph, which is based on actual observed frequency phenomena and recorded over the years by my department, we would not be able to do a good job. We do match this carefully.

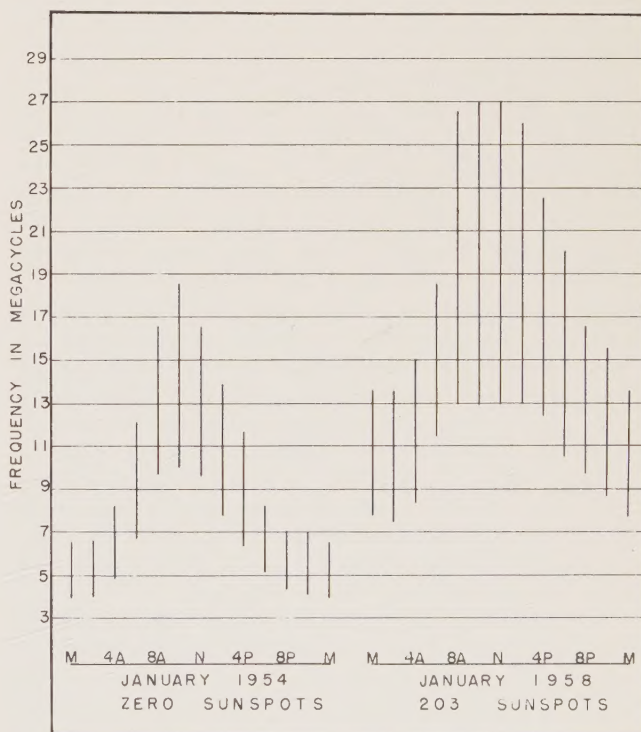


FIG. 1

Chart showing how the frequency requirements between New York and Central Europe change with the ups and downs of the sunspot cycle.

1. An informal talk given by Mr. Nelson before the Board of Directors of the Foundation for the Study of Cycles, August 12, 1960.

2. Mr. Nelson is Propagation Analyst for RCA Communications, Incorporated.

We are at the present time half way down towards the next low part of the sunspot cycle which is going to come approximately 1963 or 1964. We are already arranging our frequencies, antennas, and transmitters so that the frequencies we use at that time will match up with Mother Nature's requirements. This activity is due to a study of cycles—the sunspot cycle.

Also the sunspot numbers vary quite a bit from month to month so that even on a short term basis, we must watch our frequencies because they can change considerably.

As an example, today (August 12, 1960) we are working on the lower part of the high frequency spectrum due to the fact that we are having a moderate magnetic storm. (During magnetic storms low frequencies work best.) It is not serious, but it is enough to push our frequencies down three or four megacycles below what we were working last week. The present condition was predicted three weeks ago.

Part of my job is to predict frequencies three months in advance. This forecast, on its part, is based upon the U. S. Bureau of Standards' prediction of the sunspot cycle, because solar behavior affects the ionosphere.

Another part of my job is to forecast radio weather, more properly called "radio propagation quality." To do this I make use of other cycles which I shall describe in a moment.

These radio weather forecasts are of two kinds: One is made about the middle of each month and gives a day-by-day forecast for the following month. In fact, it goes even further and forecasts radio weather by day and radio weather by night. We call this forecast the "long range forecast." The long range forecast is based not only upon the solar cycles already mentioned but also upon the cyclic movements of the planets. By using a knowledge of these planetary movements these long range forecasts are much more accurate than they would be if they merely used a knowledge of the solar cycles. Over a three-year test period my long range forecasts attained an accuracy of 75-80%. At the present time the accuracy is about 85%. Accuracy of this degree is statistically very significant. Also it is of great practical use to my company.

In addition to the long range forecasts of radio weather, I make daily forecasts that we call "short term forecasts." These short term forecasts are made at noon, Monday to Friday inclusive, and consist of six separate forecasts for the period from 4 p.m. that day to 4 p.m. the next day. (4 p.m. to 8 p.m.; 8 p.m. to midnight; midnight to 4 a.m.; 4 a.m. to 8 a.m.; 8 a.m. to noon; noon to 4 p.m.)

On these short term forecasts I have been attaining an accuracy of 90% to 93%, year

after year.

You might like to know how this accuracy is judged. My forecasts are entirely numerical, according to the scale used throughout the world (except by the Japanese). It is as follows:

1. Useless
2. Very poor
3. Poor
4. Fair to poor
5. Fair
6. Fair to good
7. Good
8. Very good
9. Excellent

It is understood that all my forecasts are ± 1 . Thus, when I call "5" it means 5 ± 1 , that is, 4, 5, or 6. When I call "6" it means 6 ± 1 , or 5, 6, or 7.

If the actual conditions for the 4-hour period as recorded by the supervisor of the receiving station are within one number of my forecast, my forecast is considered correct; if more than one number off, it is wrong. This is the so called "Satisfactory Scale." The same method is used for judging my long range forecasts. It is also the method used by the U. S. Bureau of Standards.

Radio weather is quite variable. For example, on July 29—30, at the time of our last important disturbance, radio weather, was graded 6, 4, 2, 4, 4, 6.

Incidentally, my forecast for this period was 6 ± 1 , 4 ± 1 , 4 ± 1 , 5 ± 1 , 5 ± 1 , 5 ± 1 . This forecast, with five correct and one wrong was rated 83%. My long range forecast for July issued on June 21 also predicted a radio disturbance for July 29—31.

For the production of my short term forecast there are four basic elements required:

(I) It has been well-established through the years that sunspots are related to magnetic storms; so I naturally consider sunspots. However, everybody who has attempted to use sunspots for forecasting radio weather knows that they are unreliable, and it is impossible at the present time to forecast reliably by just using this one item. Even though we bear in mind that for *nearly* every magnetic storm, we need one or more sunspots; there are cases where magnetic storms will develop with no spots on the sun at all. There are also cases where great bursts of sunspot activity come, throughout which magnetic activity is very low. So, in addition to sunspots, I consider other items:

(II) One other thing I consider is an analysis of the earth's magnetic field for the preceding 24 hours. I make the analysis from magnetic activity data sent to me daily by the Bureau of Standards. With this I'm able to detect trends of magnetic activity.

(III) Third, I study ionospheric data supplied to me daily by reports from our field stations throughout the world. With this I can detect trends relative to changes in the ionosphere.

(IV) Fourth, and most important, I use the cycles of the planets.

Of these four items I find that planetary arrangements, when combined with the other items, produce the most reliable forecasts. A radio disturbance without a significant planetary arrangement of the type in which I'm interested rarely occurs.

By putting all these items together at noon each day on a specially prepared data sheet, I am able to make a deduction relative to the quality of the radio signals for the next 30 to 36 hours.

The planetary cycles I use are seven in number. All of these events are cyclic and are timed with great accuracy by the astronomers. Details are to be found in the *Nautical Almanac*, published by the U. S. Naval Observatory. These are the cycles I use:

1. *Cycles of Heliocentric Arrangement.* Heliocentric arrangement refers to the angular separation of the planets, one from the other, as they circle the sun.

2. *Cycles of Right Ascension Arrangement.* Right ascension arrangement refers to the angular separation of the planets relative to the center of the earth. (These cycles are presently under investigation.)

3. *Cycles of Identity of Declination.* Declination refers to the degrees that each planet is above or below the plane of the earth's equator.

4. *Cycles of Change of Direction of Declination.* This means the times when the planets stop going away from the equator and start going in the opposite direction, or vice versa.

5. *Cycles of Parallelism of Declination.* This means the times when one planet is the same angular distance below the plane of the earth's equator as another planet is above it.

6. *Cycles of Equatorial and Ecliptic Identity.* These are the times when a planet has the same angular distance from the plane of the earth's equator and from the plane of the ecliptic, i.e., the plane of the earth's orbit. In the *Nautical Almanac* the angular distance from the ecliptic is referred to as "Heliocentric Latitude." These last named cycles are being used only experimentally, but seem very promising.

7. *Cycles of Nodal Crossings.* These are the times when the planets cross the ecliptic, or the plane of the earth's orbit.

As you are more interested in cycles than in radio weather forecasting as such I shall

devote the rest of my talk to a discussion of the various cycles. Also, as you are all familiar with the daily, yearly, and sunspot cycles, I shall concentrate my attention upon my discoveries relative to the association between the various planetary cycles and radio propagation quality.

Incidentally, I have found these planetary cycles to be associated with radio disturbances so uniformly that, in my own mind, I have come to believe, as a working hypothesis, that there must be a cause and effect relationship. Therefore, although these relationships have not yet been fully proved, and although their mechanism is not yet fully understood, I feel justified in using the word "effect" in a tentative way and I trust you will so understand it, even though I leave off the quotes in the rest of this presentation.

I. Cycles of Heliocentric Arrangement

Some years ago I noticed that a radio disturbance rarely occurred without a 0° , 90° , 180° , or 270° heliocentric arrangement of two planets. ("Heliocentric" of course means "with the sun at the center." Thus, a 90° heliocentric angle between two planets means that they are 90° apart when looked at from the sun; or that imaginary lines connecting the center of these planets with the center of the sun, make an angle of this size, if looked at from above.) However, these arrangements often occurred without a corresponding radio disturbance. In fact, most of them were duds. It just meant that these angles were important, but that there was something else to it. So I carried it still further and I found out that a *grouping* of planets in these combinations had a greater "effect" than single isolated cases.

However, I found out that isolated cases sometimes could have a very severe effect. It could be only two planets at 90° and yet we would have a magnetic storm. I had literally to tear the cases apart to find out why.

For instance, it might just be Mercury and Mars at 90° and yet it would have a considerable effect. I had to find out why this case was different from the last time Mercury and Mars were 90° apart. And I came upon a very important thing, which was that there is often also present a harmonic relationship among the other planets during these cases. I found that these harmonics are tied to one of my base angles which is 90° . Simply by dividing 90° by 2, 3, 4, 5, and 6, I got 45° , 30° , $22\frac{1}{2}^\circ$, 18° , and 15° . These angles, and these angles $\pm 90^\circ$ or 180° , became my harmonics.

I also found that 30° , 60° , and 120° angles were stabilizing by themselves, but if they occur as part of a harmonic they are not.

My rule for using this system, without too much confusion, is that these contacts must be made almost simultaneous in time. That is the most important discovery I have made regarding harmonics in the past five years, the time interval. For instance, if Mercury and Mars—since I started to talk about them first, we will continue with them—are 90° at noontime, and Venus is connected to Mars at 18° at 6 o'clock in the afternoon, which is 6 hours later, nothing important happens. It must be closer than that. If at noontime, I find that Venus is 18° , plus or minus an hour, of Mars, the earth is 45° away from Mars and Jupiter is 18° away from Earth, I get a standard harmonic build-up, one connected with the other simultaneously in time. That seems to be the crux of the whole thing. A two or three hour differential in time of contact will nullify the effect of the configuration.

I work to $\pm 6'$ of arc; that is, to a tenth of a degree.

To find these exact times requires a great deal of work. I have 36 variables to analyze every day—36 different angles that are in existence at any given moment and I must find out which of these angles are harmonically related. With the calculators at my disposal in the office I have found a simple way to do this.

With these heliocentric angles I have been able to account for nearly all of the radio disturbances of any importance during the past 10 or 12 years that I have investigated. But in about 99% of the cases there must be 0° , 90° , or 180° angle for a disturbance to exist. The disturbance might start a day before that combination, or a day afterwards.

A few disturbances did not conform to that schedule, so I investigated very carefully and found out that just a series of very precise harmonics well-timed, can also do the job. I have only a few cases like that, but it definitely boils down to those significant angles in the harmonics. To simplify my technique, I move all the planets back in segments of 90° until they all fall between 0° and 90° . That reduces the number of angles I have to study by 4, one fourth.

For example, let's talk about an 18° angle which is $1/5$ th of 90° . From 90° back to 72° is also 18° . From 90° to 108° is 18° . You can take 180° and add 18° to it or take 18° away from it and you still get the same effect as you would get with an 18° angle. That's why I can move all of the angles back into one quadrant of the circle. It was a lifesaver for me when I found I could do that. It saved a great deal of time.

So much for the heliocentric position of the planets.

II. Cycles of Right Ascension

For the past several years quite a number of people have been asking me to investigate the geocentric angles (right ascension) of the planets because they thought that these angles might have something to do with it. As I had my hands full with my own method, I did not get around to it as much as I would like. I would just make a brush at it now and again but I did not get very far. With the right ascension angles of the planets, which is the angular relationship around the earth, I could not find anything significant except in a few cases.

That is the gist of where I stand today in this work. I am presently trying to incorporate geocentric right ascension into this program. As yet, I have not found out how to use these cycles reliably. I have a reputation to defend and I cannot afford to take chances with uncertain techniques. When you work with something you are not sure of you take chances. Until I learn more about the subject I am going to leave it alone. However, I believe there's a pretty good chance that there is something there. (Editor's note: Mr. Nelson has informed us that since the time of the above address he has discovered a way in which these cycles can be used usefully and dependably. He uses the 0° , 90° , and 180° angles. The 30° , 60° , and 120° angles have not yet been investigated.)

III. Cycles of Identity of Declination

I then investigated declination, which is the angular distance the planet is north or south of the earth's equator. When I did that I struck pay dirt immediately. I found that when any two planets had the same declination, it was important.

IV. Cycles of Parallelism of Declination

I also found that if any two planets had the same declination on either side of the earth's equator it would be the same as if they were on the same side of the earth's equator. I added this information to my regular technique and gained some improvement in my work.

V. Cycles of Changes of Direction of Declination

Since July 1959, I have discovered something else which is vitally important: that is when a planet changes direction in declination, it has a very pronounced effect. The day that the planet changes direction is the day that the magnetic storm starts or intensifies. I

have a significant number of cases recorded.

I will list some major magnetic storms that took place with a change in geocentric declination. We have the famous storm of February 11, 1958, which was written up in the *New Yorker* magazine. I was given credit for being the only forecaster having predicted that disturbance, though I did not predict it to be as severe as it turned out to be. Had I known then what I know now, I could have predicted it perfectly.

Jupiter on February 11 made a change in direction of declination. Radio signals dropped to quality 1. Magnetic activity became intense. To me, it was just amazing that this should work so accurately so I decided to check other cases.

On July 9, 1958, we had a very severe magnetic storm that I failed to predict because I had no 0-90-180 degree angle that day. Incidentally, it was with this disturbance that I found the effect of the 15- and 18-degree harmonics. On July 9, 1958, Neptune changed its direction of declination. There were also several 15- and 18-degree harmonic contacts involving Venus, Mercury, Uranus, and Saturn at the same hour of the day or close to an hour.

July 15, 1959, last year, we had another very severe magnetic storm. I found that on July 15, Jupiter changed direction of declination again just as it did on February 11, 1958. The earth on that day was precisely 18° ahead of Saturn and 30° ahead of Venus, which gave me the harmonics I needed.

July 14, 1960, last month, Mercury made a change in direction of declination about 36 hours before it came into 0° contact with the earth. We had a severe magnetic storm from the 14th to the 17th accompanied by a complex grouping of planets. During this combination, we had Mercury and Saturn 0° on the 14th, Venus and Neptune 90° on the 15th, Mercury at 0° with Earth on the 17th. And Mercury and Mars had the same geocentric declination. These changes of direction cases are important but by themselves I doubt if they will mean very much. They do not happen very often, only a few times a year. They appear to intensify a magnetic storm.

Let's look at some major magnetic storms that took place with a declination parallels in which two planets have the same geocentric latitude, either on the same side or on opposite sides of the equator. We have September 2, 1957. Venus and Mars were in parallel and also Mercury and Jupiter. This took place very close to the earth's equator. There was a great aurora and extreme magnetic storm at that time. September 12, Mercury and Mars were parallel near the equator also with a severe magnetic storm. April 25, 1960, Venus and Mars were parallel. April 28, Mercury and Mars,

parallel; May 7, Mercury and Venus, parallel. All were accompanied by a severe magnetic disturbance. These are, of course, merely examples.

So I have successfully worked into my system a geocentric method to support my heliocentric method and it is a most important improvement.

IV. Cycles of Equatorial and Ecliptic Identities

On the geocentric side there is one other important item that I have discovered, particularly with Mercury and Venus. When either of these planets are the same number of degrees north or south of the earth's equator, as they are from the ecliptic, there is a rather strange type of parallel.

As an example let us say that Mercury is 3° from the ecliptic the same day as it is 3° from the equator. When this takes place in timing with strong heliocentric contacts, it brings the radio signals down to a quality of 1 or 2 instead of what would probably be 3 or 4. For example, Mercury did this twice in April this year, once on the 11th, and again on the 30th and both dates had severe magnetic storms.

I find the same effect for Venus measured from the earth's equator and Venus measured from the ecliptic. I have found cases of Mars doing the same thing. On September 6, 1957, Jupiter did that same thing. We had some exceptional magnetic storms during September that year.

V. Cycles of Nodal Crossings

I have found the planetary nodal points to be very important. Saturn and Jupiter are approaching their nodes now (August 12, 1960) and on November 4, 1960, Jupiter will pass through its descending node. (Editor's note: November 4th marked the beginning of a prolonged magnetic storm.)

Saturn will pass through its descending node on April 5, 1961. On April 16, Jupiter and Saturn will be zero degrees heliocentric angle and only 21 minutes of arc from Saturn's node.

This particular contact may be the most powerful cycle in the solar system. It happens but rarely. The sidereal period of Saturn is 29.45772 years and that of Jupiter is 11.86223 years. (The sidereal period of a planet is the time it takes to go around the sun, on the average.) Perhaps someone blessed with a mathematical mind would like to figure out how often this happens. The same technique can be applied to any two planets and may reveal some interesting cycles.

Note that in this case we are not interested in the synodic period of any two planets but rather how often do any two planets have similar relationship at the same point in their heliocentric orbit. (The synodic period of two planets is the time from one conjunction to the next. A conjunction occurs when two planets have the same heliocentric longitude.)

Now I want to make it clear that I have no theory as to why these things are so. In fact I do not even claim to have scientific proof to back up the accuracy of what I have been telling you. I am merely giving you the benefit of my *observations*. Some of the associations I have noticed may be mere coincidence and may not stand up in an objective statistical study. But it is hard for me to believe that all, or even the bulk, of the associations I have observed are mere coincidences. If they are, how can I make forecasts based on these associations and have them come right so often. How is it that my forecasts are better year after year as I integrate these new observations into my system?

I would welcome a truly scientific enquiry into this whole field. Such an enquiry might turn up some things I have missed, or prove that some of the associations I have observed are not always dependable. Nothing would please me better. All I want is to make the best possible forecasts of radio weather. As of now my forecasts are an *art*; the sooner they can be made a *science* the better for all concerned.

I think I have given you a pretty good resume of my system and I'd like to make one more final statement which I consider important, before I close off for the question period, and that is, after spending over 12 years of my life in the study of sunspots, magnetic storms, high frequency radio, and research of planetary arrangements, it is only natural that I should form by this time some rather definite conclusions. I'll give you two of them.

I do not pay much attention to *isolated* planetary events. I find that to be effective there must be a *grouping* of events, both heliocentric and geocentric, occurring in *close* time sequence.

I am thoroughly convinced that the heliocentric configurations of the planets as they go around the sun play a prominent part in the development of sunspots. I have seen some very outstanding cases. It is quite possible that there is also some dynamic force taking place in the sun itself which might be necessary. I have watched this for so many years now that I am quite sure that the procedure works.

I would also like to say that it looks to me that in order to get extremely severe magnetic storms, we need to have the sun become active by the collection of planets around it, which are tied to the earth by geocentric combinations as well. In other words, the tying of the planets to the earth geocentrically while important heliocentric combinations exist, brings in the most severe magnetic storms. There have been six or seven extremely severe disturbances in the past few years and that characteristic showed up in every one.

Fortunately for my industry we do not have more than three or four really bad magnetic storms a year. Most of the magnetic storms are not so severe but that we can work through them by know-how techniques we have developed such as relays, changes in frequency, increased power, and matters of that kind. We have done such a job in overcoming magnetic storms that now it would be only the extremely severe ones that would cause us any trouble. The improvement in the communications art during the past ten years is truly remarkable to the extent that magnetic storms which twelve years ago would have stopped our radio circuits all night now have not much more than an annoyance value. And that speaks very well for the engineers working for this industry, not only in RCA Communications, but in all the branches of electronics there has been tremendous progress.

Now we are going into satellite communications, and I suppose somebody designed that just especially to put me out of business. When we go into satellite communication we won't need forecasters any more. There won't be any disturbances. But as I said to somebody else on this score, there'll probably be something else turn up which we haven't even thought about yet!

THE SO-CALLED EIGHTY-YEAR SUNSPOT CYCLE

BY EDWARD R. DEWEY

ABSTRACT

Study of all available relative sunspot numbers (1700—1960) shows that the so-called 80-year sunspot cycle is, over this period, more nearly 88-years long.

This fact suggests the need of re-examining earlier work which indicated a somewhat shorter length.

The way in which this re-examination should be made is indicated.

A cycle about 80 years long in solar phenomena has been variously alleged. For example Clough traced an 83-year cycle in the variation in the length of the sunspot cycle from A.D. 400 through A.D. 1932.¹ Lane's periodogram analysis of relative sunspot numbers, whole disk, 1749—1952, showed marked strength at a little over 84 years.² More recently Gleissberg has traced the cycle back to A.D. 290 and finds its length to be about 79 years.³

If Clough had counted one more inflection point as epoch he would have obtained a length quite close to that obtained by Gleissberg. Conversely, if Gleissberg had omitted one somewhat controversial epoch he would have obtained results closely approximating those of Clough and Lane. If he had omitted a second also controversial epoch his length would have turned out to be about 89 years.

Regardless of what should or should not be considered an epoch, and hence whether the length of this major undulation is about 79, 83, or 89 years. It seems clear that there has been, throughout recorded history, a tendency for solar phenomena to vary in a cycle of this general order of magnitude which, on its part,

is no more irregular than the 11.1-year sunspot cycle itself.

Dates of sunspot maxima are now continuously available from 300 B.C.,⁴ but data relative to the actual number of sunspots are available only from 1700.⁵ Variations in the *length* of the sunspot cycle—if one can select the proper number of epochs—will therefore give a better estimate of period. But estimate of period based on all actual year by year values, i.e., *strength*, may provide corroborative evidence, even though only 260 years of data are available. Such an estimate is the purpose of the present study.

There is reason to believe that the relative values of sunspot numbers are more accurately expressed in terms of the logarithms of the actual number plus a constant of 22 (the Sunspot Number Constant) than by the actual numbers themselves or by the logarithms of the actual numbers.⁶ In view of this fact, the logs of the actual numbers plus the Sunspot Number Constant were used for this study. They are charted in Fig. 1, curve A. The so-called 80-year cycle is visible by inspection.

To obtain its length as best one can over

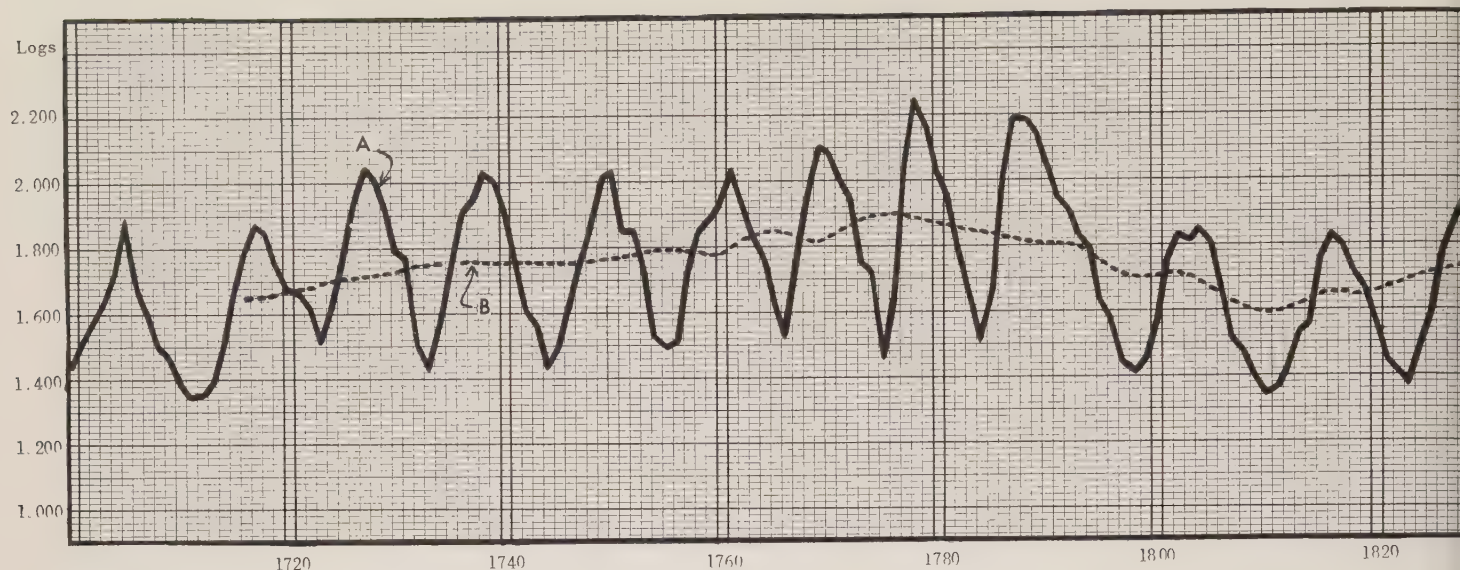


FIG. 1

Fig. 1 Sunspot numbers, plus the sunspot number constant, 1700—1960.

Curve A Logs of relative sunspot numbers, whole disk, plus the sunspot number constant of 22, 1700—1960.

Curve B A 33-year moving average of the values plotted in Curve A.

so short a period of time the values are smoothed by a 33-year moving average. Such a moving average will eliminate the dominant 11-year sunspot cycle ($3 \times 11 = 33$) and minimize any cycles from 30 to 40 years in length as well. (Lane found a 36-year cycle 1749—1952 in the *strength* of the sunspot cycle.² Clough identified a 37-year cycle A.D. 330 to A.D. 1930 in the *length* of the sunspot cycle.¹) The 33-year moving average is plotted in Fig. 1 as Curve B. It shows turning points and intervals as follows:

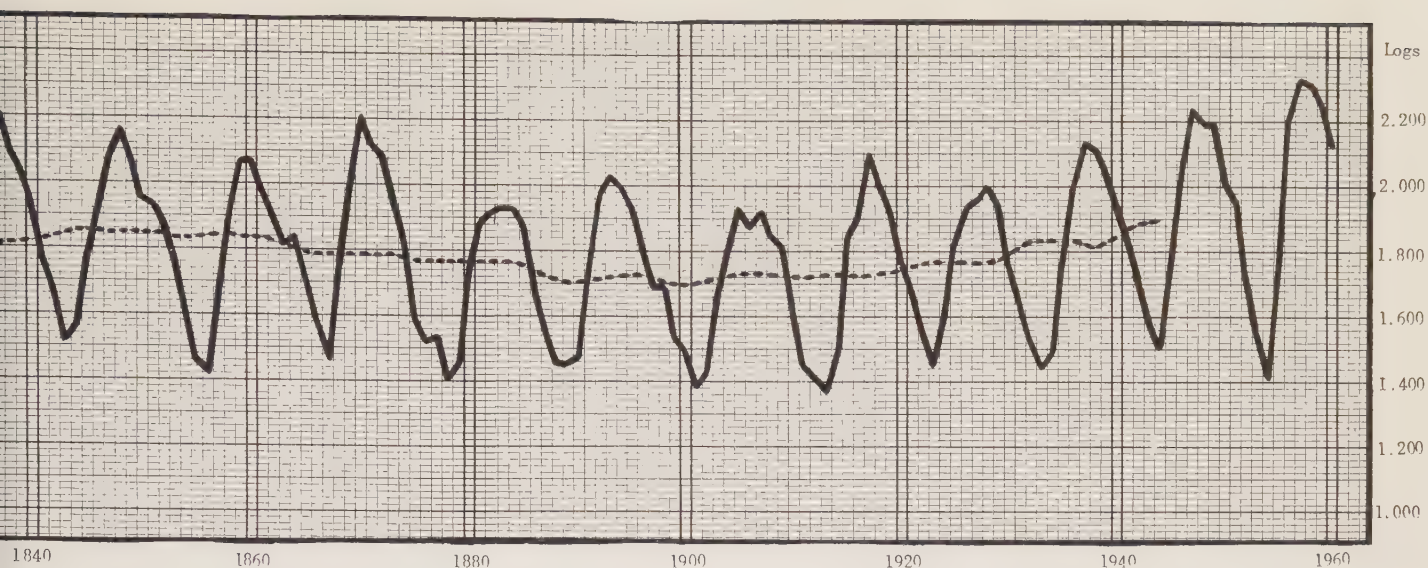
Turning Points	
Low	High
1716 or earlier	1776
1810	1845
1900	1944 or later
Intervals:	
Low to Low	High to High
94 years or more	69 years
90 years	99 years or more
Average:	
Low to Low	High to High
92 years or more	84 years or more
Grand Average:	
88 years or more	

Naturally, over so short a period (relative to the length of the cycle), and with a cycle that varies so much from epoch to epoch, these results are not conclusive. They do however suggest the desirability of re-examining Clough's and Gleissberg's work, using the 1,000 additional years of data⁴ not available to Clough (600 years of which were not available to Gleissberg either), and using new techniques not available to either of these workers at the time their work was done.^{7,8}

This cycle is an important one. Clough finds synchronous terrestrial cycles (1) in the frequency of auroras, (2) in the frequency of severe winter, (3) in the stages of the River Nile, (4) in the growth of sequoias, and (5) in the frequency of Chinese earthquakes.¹ The exact length is especially important because, if we could be certain of it, this statistic might give weight to one or another of several different theories regarding cycle cause.

The problem needs further study. The best approach would seem to be that of Clough. This approach would study the cycle wherever found to see first, if all these cycles really have identical wave length and, if so, to see what that wave length is.

The Hoskins Time Chart method would be best for determining the proper sunspot maxima to use as epochs, and probably best for determining the epochs in the other data as well.



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THE ENDOGENOUS 7.6-MONTH (33-WEEK) PERIOD*

BY J. H. DOUGLAS WEBSTER

This period has a wide range in nature and life, and only a brief synopsis of observations and theory is possible here.

A. IN DISEASES

In modern medicine Brownlee was the first to trace this period: in peaks of *influenza epidemics* in England from 1890 to the pandemic of 1918-19. When the period or a multiple fell due in the summer months, as a rule there was no epidemic. In this type of period therefore there may be "missed" periods. The period's length varied occasionally, as to 34 1/2 or 35 weeks, from 1900 to 1901, or as 32 weeks from 1915 to 1916; also half-periods become evident, as in the minor peaks at 1 1/2 periods from 1897 to 1903, and as in the pandemic half-period peaks of July and November 1918 and March 1919. (Brownlee, 1919, 1923; Webster, 1939.) The strong seasonal trend obscures this period.

2. Recurrences of *cancer and other neoplastic diseases* show this periodicity (Webster, 1938, 1940, 1942-3, 1952). This was first recognized in a patient who presented herself with very early breast scar skin recurrences, of only a few weeks' duration, after the previous signs of disease had completely disappeared following Roentgen therapy. This, in December 1937, was her fifth set of similar recurrences at approximately 33 weeks' intervals between the earliest signs of renewed activity. Units, halves and multiples of the period have been traced in the case-histories of well over 1,000 cases of cancer, leukaemia, Hodgkin's disease and other neoplastic diseases.

3. In relapses of *Tuberculosis* in many sites a similar periodicity has been noted

(Webster, 1943). Further unpublished study at the Kalmar County Sanatorium at Målilla, Sweden, of over 40 pulmonary case-histories has confirmed this observation. A number of them had had 7-year recurrences.

4. In the Psychoses this period is evident in recurrent cases with clearly defined normal and abnormal phases. (Webster, 1942, 1951 and in a book on the subject which it is hoped will appear before long.)

5. A number of *metabolic, allergic and infective* states also may have cyclical relapse tendency (Webster, 1942). Examples have been traced in records of gout, allergic food and other states, and some recurrent infections as furunculosis, iritis and appendicitis.

In this variety of illnesses the main common factor likely to be rhythmical is each patient's power of resistance to noxious agents, bodily or mental. This "resistance" factor calls for much further research, homeostatic and psychological.

B. IN NORMAL GROWTH AND DEVELOPMENT: IN CREATIVE CYCLES

Growth and development follow in the main an orderly pattern in which many stages and turning-points, and some "critical periods" may be defined. Well-dated by age the studies of Ch. Bühler, Piaget, and Gesell give many indications of forward steps which may be related to a time-pattern based on the 32 or 33-week period, and its sub-divisions and multiples. By the eighth week (1/4-period) the embryonic growth-impetus has led to the inch-long foetus with 90% of its organs formed; with the 16th week (1/2-period) foetus the skin is sensitive, and movements begin; at full-term, birth is at one period from foetal formation. In infancy, at four months (1/2-period) the alpha-wave first appears in the motor cortex and occipital leads, as visual

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powers rapidly develop. Dentition, speech, crawling, and walking, also ability to make social contacts, follow a similar step-like pattern. "Critical periods" have been noted by Bowlby in childhood in the growth of social responses; and by Kretschmer as "puberty-crises" leading possibly later to schizophrenia. Studies of growth-periods in later life may be made along lines suggested by Ch. Bühler's "Lebenslauf," and Rümke's analyses of life-phases and "Leventijdperken."

Exact dates of composition are given in quite a number of biographies and Chronological editions of the works of poets, musicians and other creative workers; and the study of periodicity in this field has been followed out in over 60 creative life-histories. A striking pattern based on the 7.6-month (33-week) mean period has been found. As an example from Walter Scott it may be noted that he began the *Border Minstrelsy* in August 1800, and ended Parts I and II in November-December, 1801 (2 periods: 15-16 months). At the age of 42 he resumed the writing of *Waverley* (and planned the *Lord of the Isles*) in September-October, 1813, and finished *Waverley* in one period in May 1814. He had a number of 7-year periods (11 of the 7.6-month one), as has already been noted by Swoboda, Pern and others. (So far the writer has published only a brief abstract of this work on periodic creativeness, which has been confirmed by studies of periodic dreams and memories. To be published it is hoped more than in the abstract of 1943.)

C. THEORETICAL

The period is in the main, endogenous, as resistance to disease is genetic, though modifiable by external factors. The periodic

aspect is largely obscure, though, as with the better understood reproductive cycles, endocrine gland activity and repose must play a large part, as in hibernation. The pituitary, thyroid and adrenal glands are the most likely to be involved, and some observations of their varying activity on this periodic basis have been made. Much physiological research is needed. This period is endogenous also because very few correspondences have been traced between illness onsets (or epidemics) and the period of minor sun-spot activity of similar length, discovered by R. Wolf of Zurich in 1859 — 100 years ago. This period still continues, though often obscured by other cycles. (Wolfe attributed it to the synodic period of Venus to Jupiter and Saturn with the sun.) Schuster's and Abbot's 20 or more periods in solar activity are, almost all, multiples of the 7.6 one, as are the meteorological ones of Brunt (1919) in Greenwich Temperatures. Long ago, geologically speaking, the period must have had its beginning in periodic solar flares, etc. which altered the meteorological environment: but it seems now a genetic memory or an archetype (Jung). The period is not a sine-wave, but a "release-reaction" (Sherrington) or "relaxation-oscillation" (van der Pol). It was known to Pythagoras (as Synesius wrote about 400 A.D.) and to Hippocrates, as an *axiom* on children's diseases shows, and he knew the half-period of 120 days. Incidentally the period is close to the Golden Mean or Section of the year; as the 7-year period is close to the Golden Mean of the 11.4-years solar cycle. Pacioli, Kepler, and Zeising would be interested in this extension of its range—already known to be a wide one in nature, life, and art.

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THE SUNSPOT NUMBER CONSTANT

BY EDWARD R. DEWEY

The investigator of short term cycles in sunspot numbers is faced with a unique problem. The short term variations are much smaller at the bottom of the main (11.09-year) cycle than at the top (Fig. 1). Any study using absolute values or absolute (arithmetic) deviations from any trend will, therefore, *minimize* the short term cycles during the trough years. On the other hand the *percentage* variations are much larger at the bottom of the main cycle. (See Fig. 2, where the values in Fig. 1 are plotted on ratio scale.) If

one studies the logarithms of the numbers, or the percentage deviations from any trend, the short term cycles at the trough of the cycle will be exaggerated.

The solution is to assume an additional number of sunspot numbers. The number chosen will, of course, be the number required to make the percentage variations from trend as nearly as possible the same at crest as at trough. This number turns out to be 22. Inasmuch as this number is added to *all* sunspot numbers it will have no effect on the length

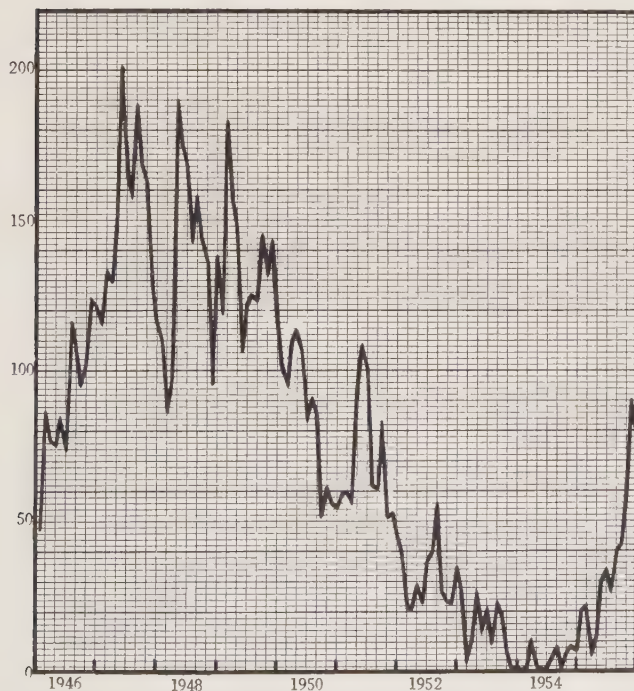


FIG. 1

Relative Sunspot Numbers (whole disc), January 1946—December 1955, plotted on arithmetic scale.

Note how small are the absolute fluctuations at the trough of the cycle.

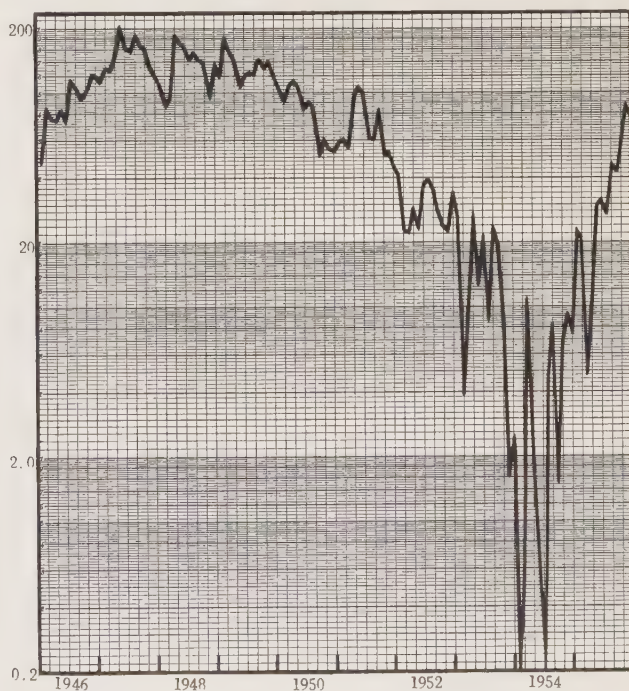


FIG. 2

Relative Sunspot Numbers (whole disc), January 1946—December 1955, plotted on ratio scale.

Note how large the *relative* fluctuation at trough of the cycle.

or phase of any cycle discovered.

Fig. 3 shows, on ratio scale, the actual relative sunspot numbers charted in Fig. 1 and Fig. 2, plus the constant of 22. It is clear that the fluctuations are more nearly of the same order of magnitude.

The method used to determine the constant was as follows:

The average number of relative sunspot numbers for all 20 years of high was calculated. It proves to be 103.7. The average deviation of the monthly averages from these annual averages was calculated. It proves to be 17.4 or 16.3%.

The corresponding numbers for the 18 years of low (ignoring 1810, which had no spots) are 6.3, 3.9, and 61.9%.

The two percentages of 16.3 and 61.9 can be brought into line by adding 22 to both the highs and lows. The absolute value of the deviations, of course, remains the same. If we do this we have—

$$\frac{17.4}{103.7 + 22} = 13.8\% \quad \frac{3.9}{6.3 + 22} = 13.8\%$$

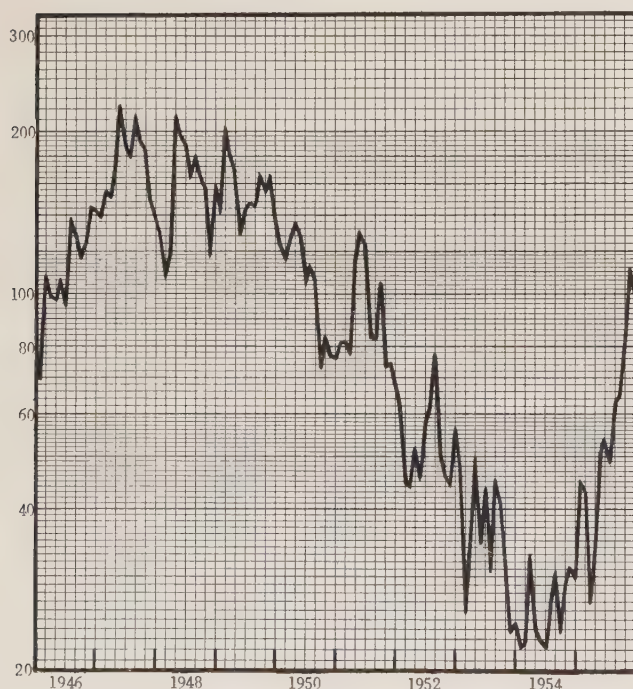


FIG. 3

Relative Sunspot Numbers (whole disc), January 1946—December 1955, plus the Sunspot Number Constant of 22, plotted on ratio scale.

Note that now the relative fluctuation at peak and trough are of the same order of magnitude.

Table 1 shows, wave by wave, the various actual numbers, deviations, and percentages. Table 2 shows, wave by wave, the same information after adding the constant of 22. The improvement is obvious.

I suggest that we call this value, 22, the sunspot number constant.

TABLE 1

Average Sunspot Numbers, Average Monthly Deviations, and Average Percentage Deviations, for Years of High and Years of Low, 1749—1960.

Year of High	Avg. Number	Avg. Dev.	%	Year of Low	Avg. Number	Avg. Dev.	%
1750	83.4	10.6	12.7	1755	9.6	5.9	61.5
1761	85.9	12.5	14.6	1766	11.4	8.2	71.9
1769	106.1	27.7	26.1	1775	7.0	4.0	57.1
1778	154.4	24.0	15.5	1784	10.2	2.4	23.5
1787	132.0	18.7	14.2	1798	4.1	3.8	92.7
1804	47.5	8.5	17.9	1810	0	0.0	00.0
1816	45.8	12.7	27.7	1823	1.8	3.1	172.2
1830	71.0	14.3	20.0	1833	8.5	3.5	41.2
1837	138.3	21.9	15.8	1843	10.7	4.1	38.3
1848	124.3	16.9	13.6	1856	4.3	2.0	46.5
1860	95.7	9.2	9.6	1867	7.3	5.1	69.9
1870	139.1	18.1	13.0	1878	3.4	2.5	73.5
1883	63.7	16.9	26.5	1889	6.3	3.6	57.1
1893	84.9	10.6	12.5	1901	2.7	2.4	88.9
1905	63.5	15.2	23.9	1913	1.4	1.1	78.6
1917	103.9	23.1	22.2	1923	5.8	3.5	60.3
1928	77.8	11.3	14.5	1933	5.7	4.6	80.7
1937	114.4	19.2	16.8	1944	9.6	6.8	70.8
1947	151.6	22.6	14.9	1954	4.2	3.5	83.3
1957	189.7	34.8	18.3				

TABLE 2

Average Sunspot Numbers Plus the Sunspot Number Constant of 22, Average Monthly Deviations, and Average Percentage Deviations, for Years of High and Years of Low, 1749—1960.

Year of High	S.S. No. + 22	Avg. Dev.	%	Year of High	S.S. No. + 22	Avg. Dev.	%
1750	105.4	10.6	10.1	1755	31.6	5.9	18.7
1761	107.9	12.5	11.6	1766	33.4	8.2	24.6
1769	128.1	27.7	21.6	1775	29.0	4.0	13.8
1778	176.4	24.0	13.6	1784	32.2	2.4	7.5
1787	154.0	18.7	12.1	1798	26.1	3.8	14.6
1804	69.5	8.5	12.2	1810	22.0	0.0	0.0
1816	67.8	12.7	18.7	1823	23.8	3.1	13.0
1830	93.0	14.3	15.4	1833	30.5	3.5	11.5
1837	160.3	21.9	13.7	1843	32.7	4.1	12.5
1848	146.3	16.9	11.6	1856	26.3	2.0	7.6
1860	117.7	9.2	7.8	1867	29.3	5.1	17.4
1870	161.1	18.1	11.2	1878	25.4	2.5	9.8
1883	85.7	16.9	19.7	1889	28.3	3.6	12.7
1893	106.9	10.6	9.9	1901	24.7	2.4	9.7
1905	85.5	15.2	17.8	1913	23.4	1.1	4.7
1917	125.9	23.1	18.3	1923	27.8	3.5	12.6
1928	99.8	11.3	11.3	1933	27.7	4.6	16.6
1937	136.4	19.2	14.1	1944	31.6	6.8	21.5
1947	173.6	22.6	13.0	1954	26.2	3.5	13.4
1957	211.7	34.8	16.4				

LAG CORRELATION AS A TOOL IN CYCLE ANALYSIS

BY RUTH V. MURTHA

Lag correlation—also called auto correlation, and serial correlation—is an important tool in cycle analysis. Under certain circumstances it can be of great help in the discovery of hidden periodicities.

It is, therefore, desirable that it be understood by cycle analysts, and that its strengths and weaknesses be appreciated.

Lag correlation is very simple:

1) First, the data are expressed as positive and negative values above or below a trend or axis.

2) Each term of the series is multiplied by itself, i.e., squared.

3) The products are added and averaged. The result gives us the first term of the "correlogram."

4) Each term of the series is multiplied by the next following term of the series. That is to say, the series is "lagged" one position and each term of the original series is multiplied by the corresponding term of the lagged series.

5) The products are added and averaged. The result is the second term of the correlogram.

6) Each term of the series is multiplied by the second following term of the series, i.e., by the series lagged two positions.

7) The products are added and averaged to obtain the third term of the correlogram.

8) Each term of the series is multiplied by the third following term of the series.

9) The products are added and averaged to obtain the fourth term of the correlogram.

And so on.

If the original series is long enough and if enough terms of the correlogram are computed,

the correlogram will disclose any periodicity (unless too short or too long), largely freed from the distorting effects of the randoms that obscured it in the original series.

The theory behind the manipulation is very simple: the randoms are equally likely to be positive or negative. When multiplied together the products of the randoms (except the first term) are also equally likely to be positive or negative. When the products of the randoms are added together and averaged they will tend to cancel each other out. If the series is long enough, the averaged products of the randoms will approach zero.

On the other hand, any true, symmetrical periodicity will not cancel out in the initial multiplication or in any lag that equals the wave length of the periodicity, or any multiple of this wave length. This is so because with such a lag, plus values will tend to be multiplied by plus values, negative values by negative values (with a consequent plus result) so that the sum of the products will be large every time the lag equals the wave length of the periodicity, or some multiple of it.

Also, when the lag equals half the length of the cycle, high (plus) values will be opposite low (negative) values so the product will be large, but with a negative sign. The result will be a large negative value as a result of the multiplication.

When randoms and a cycle are combined, the correlogram—as the chart of the successive averages is called—will thus show the cycle (or cycles) enhanced, and the randoms minimized.

All of this will be plainer if we work out some examples, using controlled data.

TABLE 1
Random Numbers Subjected to Lag Correlation

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
1	1	1	1																	
2	9	9	81	1	9															
3	15	15	225	4	135	1	15													
4	-11	-18	324	15	-270	9	-162	1	-18											
5	-20	-20	560	-18	360	15	-360	4	-160	1	-20									
6	6	6	64	-20	-160	-18	-144	15	120	9	72	1	8							
7	-24	-24	576	1	-192	-20	-480	-18	432	15	-360	9	-116	1	-14					
8	-3	-3	9	-24	72	8	-24	-20	60	-18	54	15	-45	9	-27	1	-3			
9	2	2	4	-3	-6	-24	-48	8	16	-20	-40	-18	-36	15	30	4	18	1	2	
10	-8	-8	64	3	-16	-3	24	-24	192	8	-64	-20	160	-11	144	15	-120	4	-72	1
11	-56	-56	3136	-8	464	2	-112	-3	174	-24	1342	8	-464	-20	1110	-18	1644	15	-170	9
12	25	25	625	-56	-1150	-8	-200	1	50	-3	-78	-24	-600	1	200	-20	-560	-18	-450	15
13	66	66	4356	25	1650	-56	-3120	-8	-528	2	132	-3	-198	-24	-1514	8	528	-20	-1320	-18
14	8	8	64	66	528	25	200	-58	-464	-8	-64	2	16	-3	-24	-24	-144	8	64	-20
15	-17	-17	289	8	-136	66	-1122	25	-420	-58	716	-8	136	2	-34	-3	51	-24	468	8
16	6	6	36	-17	-102	8	48	66	342	25	150	-58	-348	-8	-48	2	12	-3	-18	-24
17	17	17	289	6	162	-17	-288	8	132	66	1122	25	425	-58	-416	-8	136	2	34	-3
18	-19	-19	361	17	-324	6	-114	-17	-324	8	-152	66	-1254	25	-445	-58	1102	-8	152	2
19	-1	-1	1	-19	19	17	-17	6	-17	17	8	-8	16	-60	25	-25	-58	56	-8	8
20	-5	-5	25	-1	5	-14	70	17	-15	6	-30	-17	15	8	-40	66	-340	25	-125	-58
21	-9	-9	81	-5	45	-1	9	-19	171	17	-153	6	-54	-17	153	1	-72	66	-544	25
22	5	5	25	-4	-45	-5	-25	-1	-5	-19	-45	17	15	6	30	-17	85	8	40	66
23	-4	-4	16	5	-20	-9	36	-5	20	-1	4	-19	76	17	-68	6	-24	-17	68	8
24	-9	-9	81	-4	36	-5	45	-9	21	-5	45	-1	9	-19	171	17	-153	6	-54	-17
25	-58	-58	3364	-9	522	-1	232	5	-240	-9	522	-5	240	-1	51	-19	1102	17	-468	6
26	12	12	144	-56	-696	-4	-168	-4	-40	5	60	-9	-168	-5	-60	-1	-12	-19	-228	17
27	-8	-8	64	12	-96	-56	-464	-4	72	-4	52	-5	-40	-4	72	-5	40	-1	8	-19
28	-42	-42	1764	-8	336	12	-360	-58	-436	-9	316	-4	118	-5	-210	-9	372	-5	210	-1
29	26	26	676	-42	-1680	-1	-208	12	312	-58	-1508	-4	-274	-4	-164	5	130	-9	-234	-5
30	-12	-12	144	26	-312	-42	564	-8	40	12	-144	-58	640	-9	168	4	46	-5	-60	9
31	-52	-52	2704	-12	624	26	-1352	-42	2164	-8	416	12	-624	-58	3616	-9	462	-4	268	5
32	-4	-4	16	14	-196	-12	-168	20	-42	-58	-8	12	-11	-58	-12	-11	-58	-12	-11	-58

52	3	-	1	-	5	-	96	-	2	-25	70	-	10	-30	-	44	-11	-	4	27
53	5	5	25	3	15	-5	-25	12	60	-2	-10	-25	-125	4	20	-10	-50	35	165	-11
54	4	4	16	5	20	3	12	-5	-20	12	40	-2	-8	-25	-100	4	16	-10	-40	53
55	14	14	196	4	56	5	70	3	42	-5	-70	12	108	-2	-28	-25	-350	4	56	-10
56	-19	-19	361	14	-266	4	-76	5	-48	3	-57	-5	95	12	-218	-2	38	-25	475	4
57	13	13	169	-19	-247	14	112	4	42	5	65	3	34	-5	-65	12	156	-2	-26	-25
58	-21	-21	441	13	-273	-19	-349	14	-244	4	-64	5	-165	3	-63	-5	105	12	-252	-2
59	19	19	361	-21	-399	13	-247	-19	-361	14	266	4	76	5	95	3	57	-5	95	12
60	-6	-6	36	14	-114	21	126	13	-18	-19	114	14	-64	4	-24	5	-30	3	-10	-5
61	-13	-13	169	-6	78	14	-247	-21	-273	-13	-169	-19	-247	14	-182	4	-52	5	-65	3
62	12	12	144	-13	-156	-6	-72	19	238	-21	-252	13	156	-14	-228	14	168	4	46	5
63	7	7	49	12	84	-13	-41	-6	-42	19	138	-21	-147	13	91	-19	-135	14	78	4
64	-27	-27	729	7	-129	12	-324	-13	351	-6	122	19	-513	-21	567	13	-351	-19	513	14
65	13	13	169	-27	-351	9	91	12	156	-13	-169	-6	-78	19	347	-21	-273	13	164	-19
66	57	57	3249	13	741	-27	-1539	7	349	12	614	-13	-441	-6	-342	19	1123	-21	-1197	13
67	-12	-12	144	57	-684	13	-156	-27	324	7	-84	12	-144	-13	156	-6	72	19	-228	-21
68	10	10	100	-12	-120	57	570	13	130	-27	-210	7	70	12	120	-13	-150	-6	-60	14
69	6	6	36	10	-60	-12	-72	57	342	13	78	-27	-162	7	42	12	72	-13	-70	-6
70	-3	-3	9	6	-18	10	-30	-12	36	57	-171	13	-39	-27	11	7	-21	12	-36	13
71	61	61	3721	-3	-123	6	366	10	610	-12	-132	57	-3477	13	743	-27	-1641	7	427	12
72	-19	-19	361	61	-1159	3	-57	6	-114	10	-190	-12	-227	57	-1683	13	-2447	-27	513	7
73	67	67	4489	-19	-1292	61	9447	-3	-204	6	461	-12	610	-12	610	-57	-3676	13	-584	-27
74	10	10	100	67	-1058	-19	-304	61	976	-3	-48	6	46	10	160	-12	-142	57	912	13
75	10	10	100	16	160	67	660	-19	-190	61	610	-3	-30	6	60	10	100	-12	-120	57
76	-6	-6	36	10	-60	16	-120	68	-544	-19	152	61	-482	-3	24	6	-48	10	-60	-12
77	3	3	9	-8	-24	10	-30	16	48	68	-264	-19	-57	61	183	-3	-9	6	18	10
78	19	19	361	3	57	-8	-102	10	190	16	304	68	-1292	-19	-361	61	1139	-3	-57	6
79	-4	-4	16	19	-76	3	-12	-8	32	10	-40	16	-64	68	-272	-19	76	61	-244	-3
80	-2	-2	4	-4	8	19	-38	3	-6	-8	16	10	-20	16	-32	68	-136	-19	32	61

Total
Terms
Average

40823
80
1510

-4672
79
-59

-1705
78
-22

+6258
77
+89

+4361
76
+57

+1618
75
+22

-201
74
-3

+7160
73
+98

-4627
72
-64

-1472
71
-21

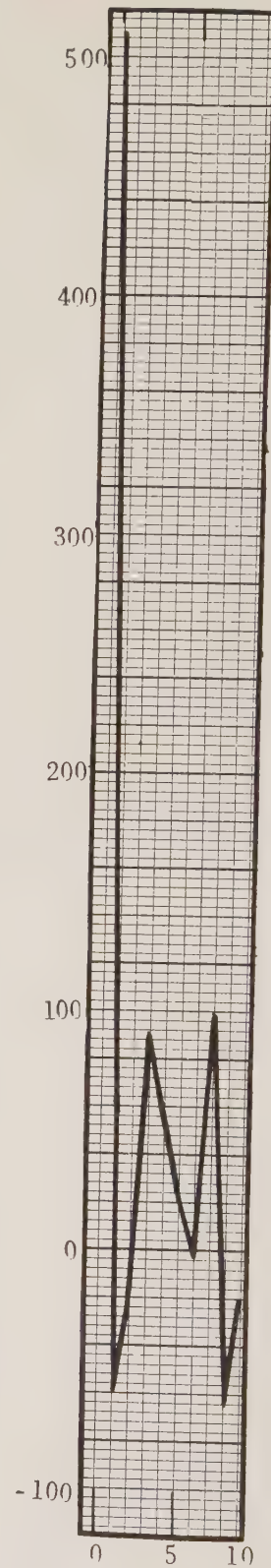


Fig. 3: Chart of
The Correlogram
Values Computed
in Table 1



FIG. 1

Random Numbers, a True Periodicity, and their Combinations

Consider first a series of random numbers as listed in Table 1, Col. A and charted in Fig. 1. These numbers are repeated in Table 1, Col. B. The numbers on each line of Col. A and Col. B are multiplied together and the product posted in Col. C. Col. C is added and averaged. The result is 510. This is the first value of our correlogram. It is very large as all the randoms are squared and are therefore positive. For present purposes ignore it.

Now let us lag the series by one position as in Table 1, Col. D. The numbers on each line of Col. A and Col. D are multiplied together and the product posted in Col. E. These values are added and averaged to obtain -59, the second value of our correlogram.

The process is repeated indefinitely. Col. F being lagged 2 positions, Col. H being lagged 3 positions, etc. up to Col. T which is lagged nine positions. In practice the process would be continued to get more value for the correlogram, but 9 lags are enough to illustrate the principle.

A graphic illustration of the process, for

the first twenty terms; is shown in Fig. 2, curves A—E. Curve A in Fig. 2 charts the first twenty values of our original series, with the horizontal scale enlarged three times. Curve B repeats Curve A. Curve C shows Curve A displaced or lagged one position to the right as in Col. F of Table 1. And so on.

Fig. 3 charts the 10 correlogram values on the same scale used in Fig. 1 to chart the original values.

It is obvious that the correlogram values are worse than the randoms they are supposed to eliminate. In fact the original values regardless of sign, average 16 and the correlogram values (excluding the value for zero lag) average 48, 3 times as much. However, it should be obvious that the more terms we have in our series the more the positive and negative values will tend to offset each other and the closer the correlogram values will be to zero. If we had 1,000 or 2,000 terms in our original series the correlogram values would be very low indeed.

Moreover, as we will see, the periodicities

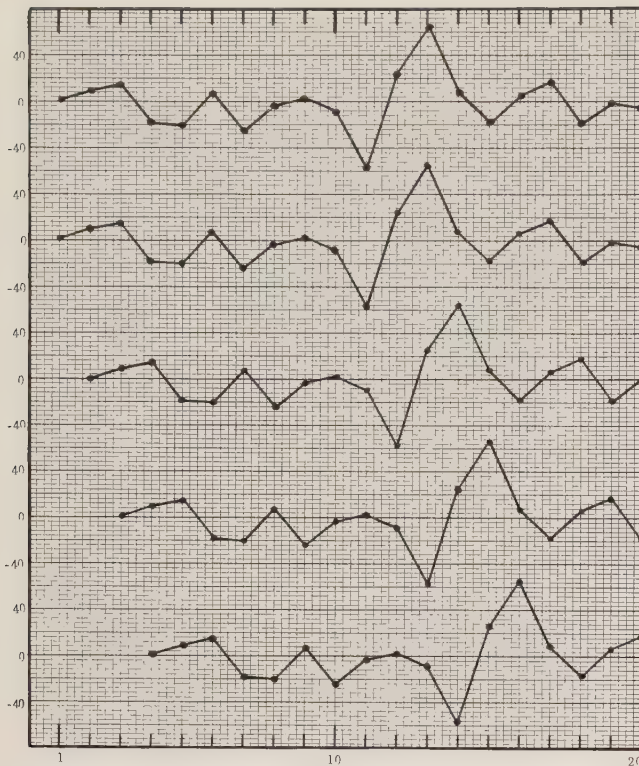


FIG. 2

Graphic Representation of The Lagging Process Using the First Twenty Terms of the Series in Table 1.

will be exaggerated; so, even with as few as 80 terms the situation is not as bad as it seems.

Before applying the lag correlation technique to a true periodicity or to a combination of a periodicity and randoms, there is a matter that should be called to your attention. When your series has been lagged one position there is nothing by which the last term can be multiplied. There are four ways to handle this situation, none of them entirely satisfactory.

1) We can, as is sometimes done, repeat the original series. That is, in the example worked out in Table 1, we could post into Col. A lines 81, 82, 83, 84 etc. the values of Col. A lines 1, 2, 3, 4, etc. (These values are 1, 9, 15, -18, etc.) Then we could continue Col. D to the 81st position, multiply it by the value in Col. A, post the product to Col. E, add the product into the total, and divide by 80 instead of by 79. In our example this procedure would change our sum of the products from -4,672 to -4,674, our correlogram value from -59 to -58. For Col. G, which is

the product of the original series and the original series lagged two positions, we would have two more products, 1×-4 and 9×-2 . This would make the sum of the products -1740, instead of -1718. Their average would be 22. And so on.

2) A second way is to assume that all values after the end of the series are zero. In our example this would make all the products after the 80th line zero. There would however be 80 terms to be averaged, so all the sums of the products would be divided by 80.

3) A third way, and the one we have chosen in our example, is to discard all values after the end of the series on the grounds that there is nothing to multiply by after this point. When you adopt this method you get your average by dividing successive sums of products by successively smaller numbers. In our example the divisor of Col. C is 80; of Col. E is 79; of Col. G is 78, etc.

4) Finally, you could use only one half of the series. In this way there would always be a suitable number by which to multiply. But of course more than half of the value of the process would be lost.

The pros and cons of these various methods will be discussed later after we see the effect of lag correlations on a perfectly regular periodicity.

Consider the perfectly regular 8-item periodicity charted in Fig. 1 as Curve B. It has an amplitude of 8. Let us lag it progressively as in Table 2, multiply, total, and average as indicated. The first 25 values of the correlogram of the 8-item periodicity are charted in Fig. 4. It is clear that the 8-item periodicity with an amplitude of 8 has become an 8-item periodicity with an amplitude of 24.

We can generalize this change of amplitude of the periodicity as follows:

For a saw tooth wave it will be the original amplitude, squared, multiplied by a constant (depending on the frequency of observation) from $1/2$ to $1/3$.

For a sine wave the constant will be about $1/2$.

If now we combine the periodicity and the randoms we will obtain the curve plotted in Fig. 1 as Curve C. Applying lag correlation to the combined figures will give us the kind of a correlogram that we would get in an actual series of figures that combined a periodicity and randoms.

In the example chosen the lag correlation technique has not helped. Our randoms and our cycle have both been increased three fold. The correlogram curve will reveal nothing about the periodicity that was not apparent in

TABLE 2

A Perfectly Regular 8-Term Periodicity Subjected to Lag Correlation

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
1	0	0	0																		
2	4	4	16	0	0																
3	0	0	0	4	32	0	0														
4	4	4	16	0	32	4	16	0	0												
5	0	0	0	4	0	0	0	4	0	0											
6	-4	-4	16	0	0	4	-16	0	-32	4	-16	0	0								
7	0	0	0	4	32	0	0	4	-32	0	-64	4	-32	0	0						
8	4	4	16	-4	32	-4	16	0	0	4	-16	0	-32	4	-16	0	0				
9	0	0	0	-4	0	-4	0	0	0	0	0	4	0	0	4	0	0	0	0		
10	4	4	16	0	0	-4	-16	-4	-32	-4	-16	0	0	4	16	0	32	4	16	0	0
11	0	0	0	4	32	0	0	-4	-32	-4	-64	-4	-32	0	0	4	32	0	64	4	32
12	4	4	16	0	32	4	16	0	0	-4	-16	-4	-32	-4	16	0	0	4	16	0	32
13	0	0	0	4	0	0	0	4	0	0	0	0	0	-4	0	-4	0	0	0	4	0
14	-4	-4	16	0	0	4	-16	0	-32	4	-16	0	0	-4	16	-4	32	-4	16	0	0
15	0	0	0	-4	32	0	0	4	-32	0	-64	4	-32	0	0	-4	32	-4	64	4	32
16	4	4	16	-4	32	-4	16	0	0	4	-16	0	-32	4	-16	0	0	-4	16	-4	32
17	0	0	0	-4	0	-4	0	-4	0	0	0	4	0	0	4	0	0	0	0	-4	0
18	4	4	16	0	0	-4	-16	-4	-32	-4	-16	0	0	4	16	0	32	4	16	0	0
19	0	0	0	4	32	0	0	-4	-32	-4	-64	-4	-32	0	0	4	32	0	64	4	32
20	4	4	16	0	32	4	16	0	0	-4	-16	-4	-32	-4	16	0	0	4	16	0	32
21	0	0	0	4	0	0	0	4	0	0	0	-4	0	-4	0	-4	0	0	0	4	0
22	-4	-4	16	0	0	4	-16	0	-32	4	-16	0	0	-4	16	-4	32	-4	16	0	0
23	0	0	0	-4	32	0	0	4	-32	0	-64	4	-32	0	0	-4	32	-4	64	4	32
24	4	4	16	-4	32	-4	16	0	0	4	-16	0	-32	4	-16	0	0	-4	16	-4	32
25	0	Etc.		-4	0	-4	0	-4	0	0	0	4	0	0	4	0	0	0	0	-4	0
26	4		Etc.		-4	-16	-4	-32	-4	-16	0	0	4	16	0	32	4	16	0	0	
27	0			Etc.		0	-4	-32	-4	-64	-4	-32	0	0	4	32	0	64	4	32	
28	4						Etc.	-4	-16	-4	-32	-4	-16	0	0	4	16	0	32	4	
29	0							Etc.		0	0	-4	0	-4	0	-4	0	0	0	-4	0
30	-4								Etc.	-4	16	-4	32	-4	16	-4	32	-4	16	0	0
31	0									Etc.		0	0	-4	0	-4	0	0	0	-4	0
32	-4										Etc.	-4	16	-4	32	-4	16	-4	32	-4	32
33	0											Etc.		0	0	-4	0	0	0	-4	0
34													Etc.							Etc.	
Total		+1920		+1280		-16		-176		-124		-1216		+16		+1152		11728		+1152	
Terms		80		79		78		77		76		75		74		73		72		71	
Average		24		16		0		-16		-24		-16		0		16		24		16	

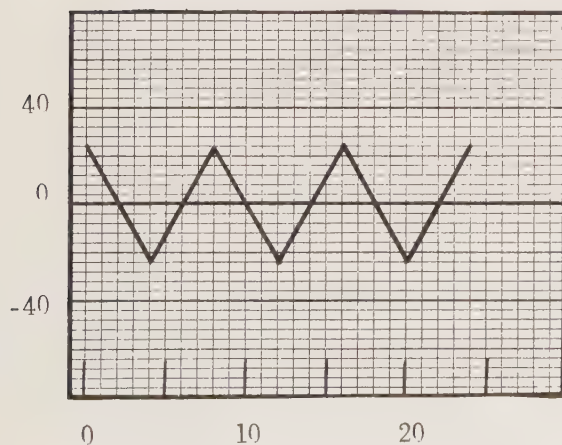


FIG. 4

Correlogram of a Perfectly Regular 8-Term Periodicity.

the original curve (Fig. 1 Col. C). But, if the original curve combining the periodicity and the randoms had been many times as long the randoms would have been reduced relative to the periodicity; and the periodicity, invisible in Fig. 1 Curve C, would have become clearly apparent.

While we are on the subject it may be mentioned that the reduction in the size of the randoms is in proportion to the square root of the length of the series. Thus if a series 80 terms in length has a correlogram with an average deviation of 48 you could cut this average deviation in half by making your series four times long ($2 \times 2 = 4$). You could cut it to 1/3 by making the series 9 times as long ($3 \times 3 = 9$), etc.

A series 1280 items long ($4 \times 4 \times 80$) would reduce the average deviation of the correlogram to 1/4. If it were 2880 items long ($6 \times 6 \times 80$) it would reduce it to 1/6 of what is produced by a series of 80 terms.

Enough has been said to show the technique. Let us now consider some of the advantages and disadvantages of the method:

Advantages

1. Lag correlation is the best method so far devised to minimize randoms relative to true periodicities, *if the series being correlated is long enough i.e., has enough terms.*

2. The periodicity will be disclosed, regardless of fractional lengths, i.e., the period does not have to be 4, 5, or 6 items long. It can just as well be $4 \frac{1}{8}$, $5 \frac{1}{10}$ or $6 \frac{7}{9}$ items long.

Disadvantages

1. The process is laborious (but of course the work can be reduced by the use of an electronic computer).

2. If there are several periodicities in the original series they will all be present jumbled together, in the correlogram. Lag correlation is in no sense selective or of any immediate aid in separating periodicities. It merely tends to minimize randoms.

3. If the periodicity is not symmetrical, the amplification of the periodicity relative to the randoms, is reduced. (See Fig. 5 for the correlogram of a lag correlation analysis of an 8-item periodicity with values of 0, -8 -5 $\frac{1}{3}$, -2 $\frac{2}{3}$, 0, +2 $\frac{2}{3}$, +5 $\frac{1}{3}$, +8).

4. Earlier I said that all four methods of handling the terminal multiplication had disadvantages.

a. If the series is cross multiplied with the original series repeated we introduce a serious distortion unless the length of the

original series is an exact multiple of the length of the cycle. Witness Fig. 6 which shows a correlogram of a series of 12 terms containing an 8-item cycle, repeated.

If the original series contained more than one periodicity, all but one of the periodicities would be distorted, unless it (or they) had periods which were exact fractions of the period that, on its part, happened to be an exact fraction of the length of the series.

This first method should therefore never be used.

b. If the series is assumed to continue as a succession of zeros, the correlogram values will approach zero. That is, there will be an important reduction of strength, both for the randoms and for the periodicities, as the successive values of the correlogram are computed.

c. If the series is assumed to end with its last value, successive terms of the correlogram will be based on fewer and fewer products and consequently will have the randoms less and less perfectly removed. Also, the periodicity will be more and more distorted.

d. The fourth method—using only half of the data—is best from the standpoint of the inherent periodicities, but it is the least satisfactory from the standpoint of removing randoms.

5. Lag correlation has only limited value if the cycle is not exactly regular as to period.

6. Lag correlation has only limited value if the cycle changes wave length.

7. Lag correlation will not reveal changes in amplitude of the cycle.

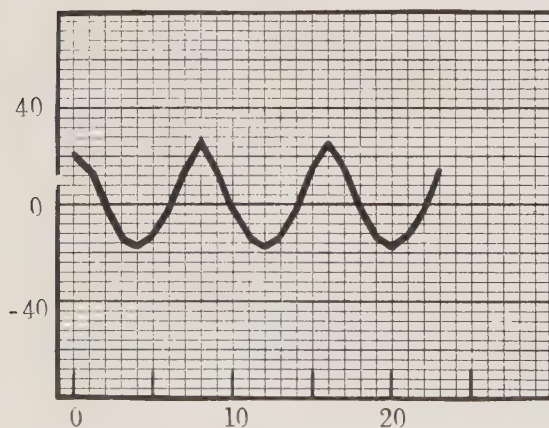


FIG. 5

Correlogram of an asymmetrical 8-term periodicity. Note that the correlogram shows a symmetrical 8-term wave, but with amplitude reduced compared to what it would have been (-24) if the wave had been symmetrical.

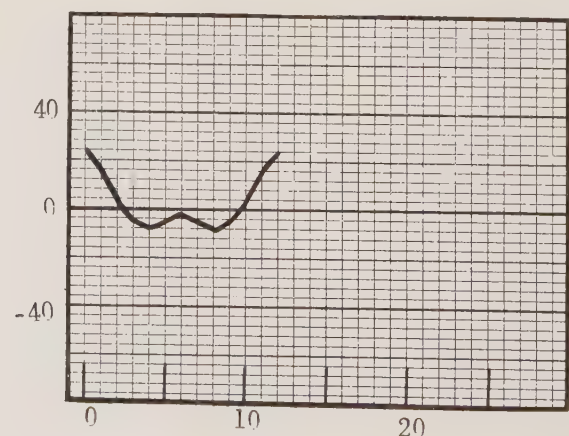


FIG. 6

Correlogram of a 12-term series containing an 8-term wave repeated one and a half times. For cross multiplication we repeat the original series but as it is out of phase we get nothing but a bad distortion.

A POSSIBLE 5.89-YEAR CYCLE IN SUNSPOT NUMBERS

BY EDWARD R. DEWEY

ABSTRACT

Relative sunspot numbers, 1749—1960, plus the Sunspot Number Constant of 22 are studied to see whether or not a 5.91-year cycle is present. This length of 5.91 years is chosen for study because of the importance of cycles about this length in terrestrial affairs.

The Sunspot Number Constant is added because short cycles in sunspot numbers cannot be studied adequately otherwise.

The result of the study was negative. No 5.91-year cycle in sunspot numbers was found. If the terrestrial cycles of this general order of magnitude are indeed 5.91 years long, there seems to be no corresponding solar cycle.

In these figures there is a hint of a cycle of fair strength that is perhaps 5.89 years long—a shade shorter than the 5.91-year cycle that was looked for. However, without the use of monthly sunspot numbers (in contrast to the annual ones used for this study) and/or the removal of other cycles of nearby lengths, it is impossible to know the length for sure, or to know whether or not this so-called 5.89-year cycle is present as a rhythm, or is merely present on the average on each half of the data.

A cycle about 5.91 years long has been found in a variety of terrestrial phenomena. It has been found in the liabilities of business failures, 1857—1953,¹ rail stock prices, 1831—1956,² combined stock prices, 1871—1956,³ industrial stock prices, 1871—1950,⁴ cotton prices, 1731—32—1949—50,⁵ pig iron prices, 1784—1953,⁶ and copper prices, 1784—1953.⁷ The cause of this cycle is unknown, but it is too long-continued and too regular to be easily the result of chance; too diverse to be easily caused by forces within the systems involved.

It has been conjectured that the ultimate cause of earthly cycles such as this one may be extra-terrestrial—either associated with

heliocentric planetary movements or arrangements or with ultra-long energy waves or cosmic waves from outer space. In either event, if there are forces timed in this way they might also affect the sun. If so, such an influence might be reflected in sunspot numbers. It therefore seems worth while to see if a 5.91-year cycle in sunspot numbers exists. The present investigation was directed towards this end.

The results are negative. It did not prove possible to isolate a cycle of this length in annual sunspot numbers, 1749—1960. This fact is an argument against the theories of ultra-long energy waves and/or heliocentric planetary movements or arrangements, although

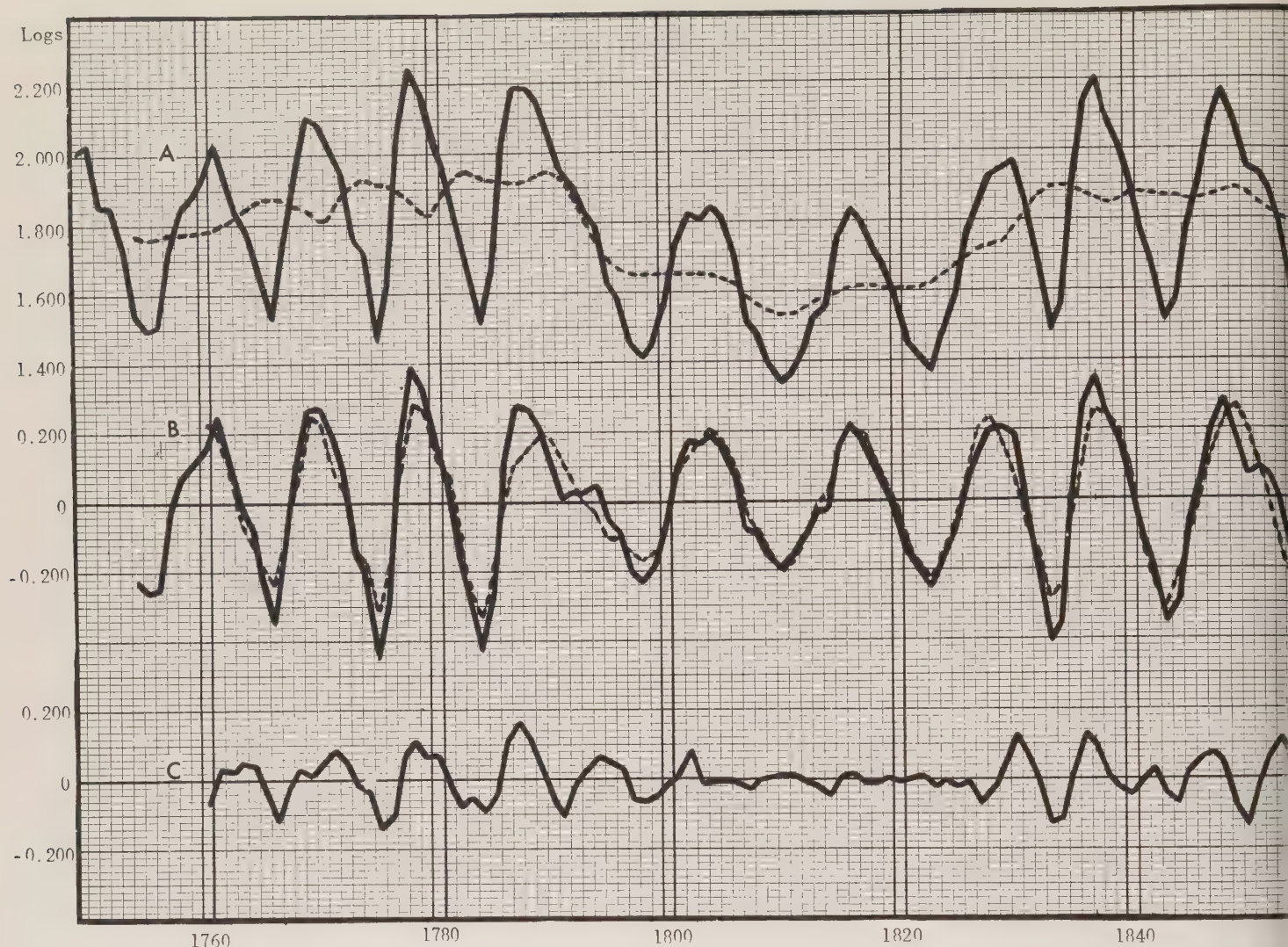


FIG 1-A Solid line: Relative sunspot numbers plus the sunspot number
 1-B Solid line: Deviations, numbers from their 11-year moving average
 1-C Deviations, deviations (1-B) from their 13-year weighted moving average

of course not a final one.

This enquiry is not the last word. It used only annual data 1749—1960. Monthly data would have been better. It did not use annual data 1700—1748 which were not conveniently available when the study was made. With more work it might be possible to effect a better elimination of what seem to be rhythmic cycles of nearby length. No attempt was made to see if a 5.91-year cycle is present in sunspot numbers, alternate cycles reversed.

The enquiry did turn up a definite hint of a cycle just a shade shorter than 5.91 years, viz. 5.89 years long. If, upon re-examination, the terrestrial 5.91-year cycles should prove to be a shade shorter also, further investigation of this 5.89-year cycle in sunspot

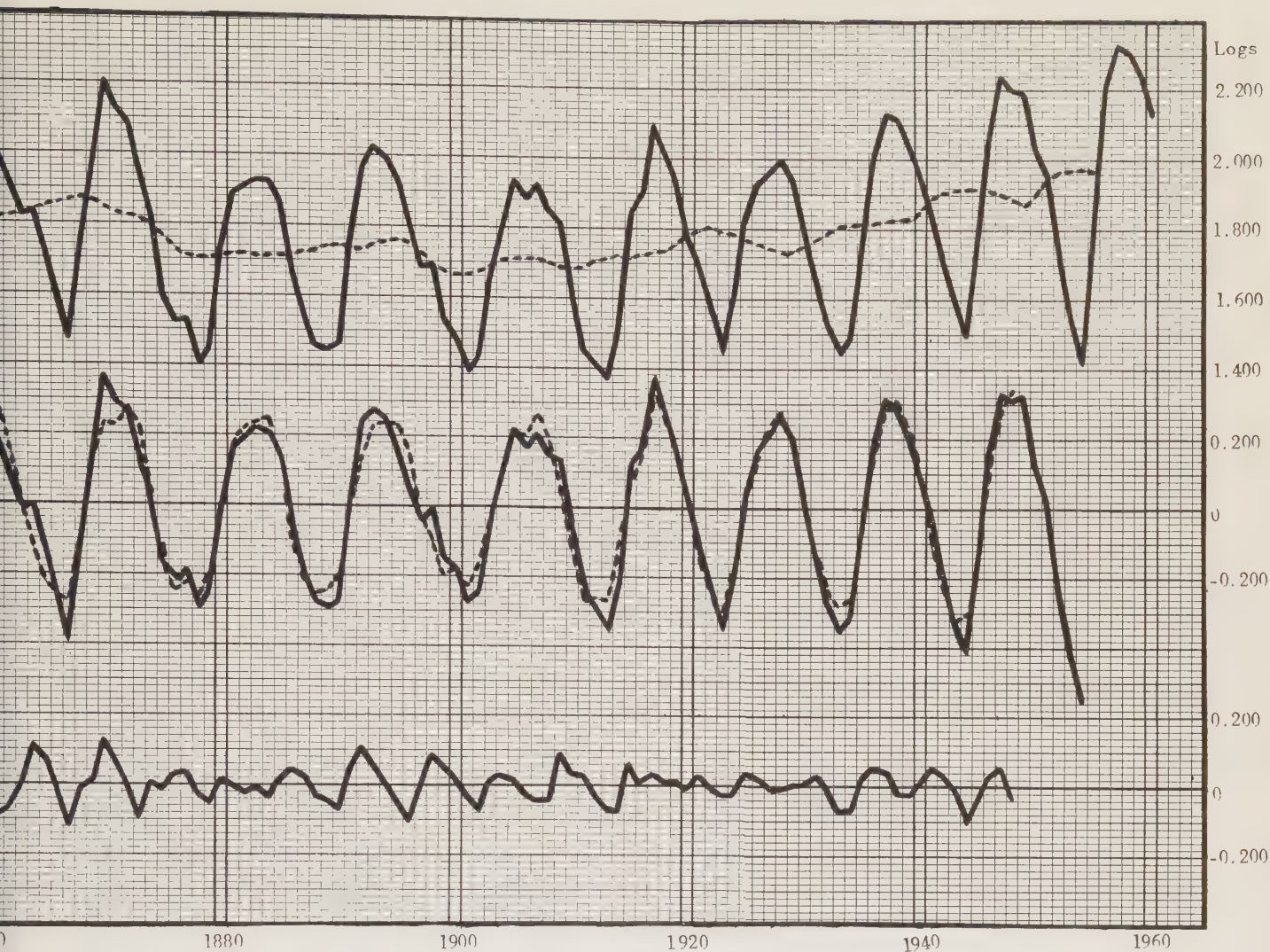
numbers should be prosecuted and the cycle definitized. In the meantime it may be noted that both the so-called 5.91-year cycle on earth and the possible 5.89-year cycle on the sun crest at approximately the same time.

It will not escape the observing reader that 5.89 years is exactly $1/5$ of the sidereal period of Saturn. At the moment there is no evidence that this fact is more than coincidence.

Analysis

The data used were relative annual sunspot numbers, whole disks, 1749—1960.

In order to make the short term fluctuations of about the same percentage strength at



1
 , 1749—1960, in logs. Broken line: Their 11-year moving average.
 line: 13-year weighted moving average of the deviations.
 ges.

the tops of the various sunspot cycles as at the bottoms, the constant of 22 was added to each of the annual averages.⁸ Logs of the result are plotted as a solid line in Fig. 1-A.

The basic 89-year undulation is clearly evident, but does not concern us here.

We first computed the 5-year moving average of these values (logs of data plus 22), and computed the deviations from this five year moving average trend. However, these deviations are so dominated by vestiges of the manifest 11-year cycle that they prove to be useless for our purposes. For the analysis of the 5.9-year cycle more powerful methods are necessary.

It was decided to use deviations from a 13-term weighted moving average designed in such a way as to fit more closely into the

peaks and troughs of the 11-year waves. For such a weighted moving average we need values expressed above and below an axis. An 11-year moving average was decided upon as the axis. This moving average is shown by the broken line in Fig. 1-A. Deviations from this moving average are shown in Fig. 1-B.

The 13-term weighted moving average of weights $-1/4, 0, 0, 0, 0, 0, +1/2, 0, 0, 0, 0, 0, -1/4$ almost exactly reproduces any regular waves that are about 11-years long and almost perfectly eliminates any waves that are about 6-years long.

The deviations of the values plotted by the solid line in Fig. 1-B from the weighted moving average are shown in Fig. 1-C.

It hardly needs to be said that as the 13-

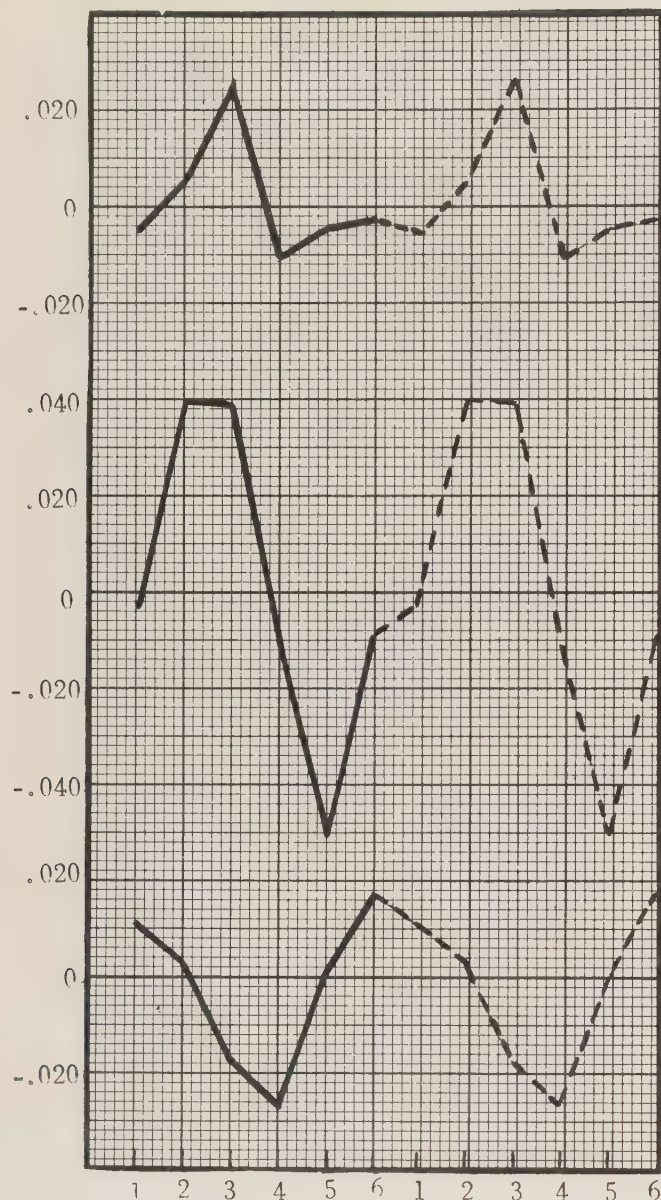


FIG. 2

Chart showing average of each 12 waves (roughly each third) of the 5.91-year periodic table. Waves are repeated in phantom. Slippage to the left of successive thirds shows that the cycle is shorter than 5.91 years long.

term weighted moving average approximately reproduces the 11-year waves, the deviations from the weighted moving average will be largely free from any 11-year vestiges. Conversely, as the 13-term weighted moving average almost completely eliminates any regular 5.9-year cycles, the deviations from the 13-term weighted moving average will show the 5.9-year cycle, if present, in full force.

It is clear from an examination of curve

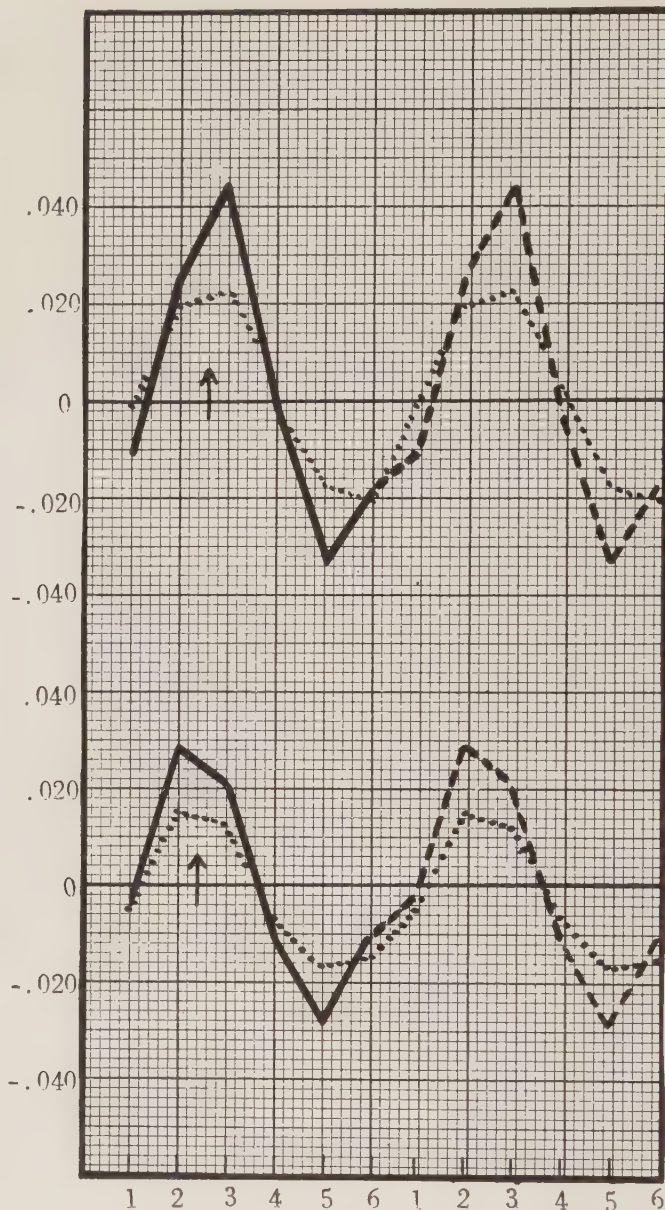


FIG. 3

Chart showing average of overlapping sections of 21 waves (to eliminate 5.625-year cycle). Broken lines are 3-year moving averages of average waves. Slippage to the left suggests a 5.89-year length. Waves repeated in phantom.

1-C that there is no 5.9-year cycle present as a visible rhythm. If there is a cycle of this length it is obscured by the 5.62-year cycle discovered by Lane⁹ and perhaps by a 5.5-year cycle as well.

The deviations from the 13-term weighted moving average are now posted into a 5.91-year periodic table which is added by thirds with the result shown in Fig. 2. It is clear that crests and troughs are present on the average

in each third of the deviations but that crests and troughs slip consistently to the left from third to third. This fact suggests that if a rhythm of this general order of magnitude is present in these figures that it is a little shorter than 5.91-years in length.

The length disclosed by the 5.91-year table added by thirds (Fig. 2) is distorted by the 5.625-year cycle. To eliminate this distortion the periodic table must be added by sections that are 21 waves long. This has been done with the results shown by the solid line in Fig. 3. The crest and trough of the second 21-wave section fall slightly to the left of the crest and trough of the first 21-wave section, indicating a length slightly shorter than 5.91 years.

To get the length of the cycle as accurately as possible we then take a 3-year moving average of each section of the table. We find that the slippage amounts to .20 years in 11 cycles or .02 years per cycle. This gives us a length of 5.91 - .02 or 5.89 years.

This length 5.89 years is probably as close

as we can come to the length of this cycle in these figures by the use of these methods.

For a more accurate determination it would be necessary to isolate and remove the conflicting cycles and to use monthly sunspot numbers, or both. This course does not seem important at this time as the purposes of the original investigation—namely to see whether or not there is a 5.91-year cycle present in these figures—has been fulfilled with a negative answer.

The first 21-wave section of the 5.91-year table is based on the year 1821.5. It shows a crest at 2.6 years after base. Crests thus come at 1824.1 and at 5.89-year intervals thereafter. This puts the current crest at 1959.57.

On the average the cycle has an overall amplitude (in logs) of .060. The amplitude is .030. This value represents 7.2% of the values we started with, namely sunspot numbers plus 22. This is a rather sizeable variation and suggests that further exploration in this area might be fruitful.

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POWER SPECTRUM ANALYSIS

Power spectrum analysis is a method of determining the strength of "power" of cycles of various arbitrarily chosen wave lengths.

Its strength lies in the fact that the cycles whose strength is determined are largely freed from the influence of randoms. Its weakness lies in the fact that the wave lengths chosen for consideration have no necessary relationship with the true wave lengths (if any) inherent in the data.

As we consider the process in detail these strengths and weaknesses will become obvious.

The Process

First of all, power spectrum analysis begins where lag correlation analysis leaves off. As stated in my article "Lag Correlation as a Tool for Cycle Analysis," the results of a lag correlation analysis are a series of figures which is plotted on a curve called a correlogram. This series—this curve—contains all periodicities present in the original figures, exaggerated in amplitude, and, if the series is long enough, with the randoms largely eliminated. Trend is already eliminated as part of the process of making the lag correlation.

The lag correlation technique modifies the inherent periodicities in other ways. It will make nonsymmetrical cycles into symmetrical cycles (but at the expense of amplitude), it will make varying length cycles into cycles of uniform rigid length (also at the expense of amplitude), and it will rephase—retime—all the cycles so that they will all crest together at position zero. If, however, any of the cycles in the original curve were of changing wave length—progressively longer or shorter, they will appear in the correlogram at their *average* length, with reduced amplitude, or perhaps even completely washed out.

Now a power spectrum analysis consists merely of making a simple Fourier or harmonic analysis of these correlogram figures or—in actual practice—the first segment of them. (As the randoms are progressively less well removed, the first—normally left hand—sections of the correlogram is "cleaner" i.e. more largely freed from randoms.) If the original series consisted of 1,000 terms the power spectrum analysis would, in practice, be limited to the first 200 or 250—at most 400 terms of the correlogram.

A harmonic analysis derives its name from sound wave analysis. Sound waves usually have lesser waves known as "overtones," or "harmonics," superimposed. Their overtones have wave lengths that are always simple unit

fractions of the length of the fundamental sound wave itself i.e., $1/2$, $1/3$, $1/4$, $1/5$, etc. Consequently cycles that are simple unit fractions of some base length (called the fundamental) are called harmonics. And an analysis of a series of figures that computes the timing and strength of cycles of these purely arbitrary lengths is called a harmonic analysis.

Let us take an example: Suppose the series of which we wish to make a harmonic analysis is 300 items (days, weeks, months) long. These 300 items constitute our fundamental, 150 items give us our 2nd harmonic, 100 items give us our 3rd harmonic, 75 items give us our 4th harmonic, etc. down to 2 items which give us our 150th harmonic.

The component cycles are of *purely arbitrary wave lengths*. Thus, in our harmonic analysis of the correlogram, if we had discarded all but 264 terms, our cycle lengths would have been 264, 132, 86, 66, items long etc. The lengths of the component periodicities depended entirely on the number of terms of the correlogram you choose to work with. They have no possible connection, except by accident, with any periodicities actually present in the correlogram or the original series.

A harmonic analysis makes another assumption. It assumes that the cycles are all smoothly undulating "sine" or "cosine" shape.

In net, a harmonic analysis fits an ideal sine (and/or cosine) curve to the data to be analyzed and notes its strength (and timing). It then chops the series in two, averages these two sections, fits a sine (and/or cosine) to the average of the two sections, and notes its strength (and timing), and so on.

In the correlogram with which we deal in a power spectrum analysis all cycles are in phase (in step) with crests at positions zero. Positioning the cycles in time is therefore not necessary. We can therefore dispense with the fitting of the sine and limit ourselves to fitting the cosine.

Having fitted cosines to the arbitrarily chosen part of the correlogram (always starting with the zero position) for each unit fraction of the length of the part chosen, we note the strength of these various cosines and chart the result on a periodogram called the spectrum.

The analysis is completed. It's value, from the standpoint of discovering hidden periodicities, is limited, unless one of the inherent periodicities happens to have a wave length equal to one or another of the arbitrarily chosen harmonics of the arbitrarily chosen segment of the correlogram. For further details see *Journal of Cycle Research*, Vol. 6, No. 2.

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